21.

NONLINEAR ELEMENTS

Earthquake Resistant Structures Should Have a Limited Number of Nonlinear Elements that can be Easily Inspected and Replaced after a Major Earthquake.

21.1 INTRODUCTION

{ XE "Energy:Energy Dissipation Elements" }{ XE "Nonlinear Elements" }Many different types of practical nonlinear elements can be used in conjunction with the application of the Fast Nonlinear Analysis method. The FNA method is very effective for the design or retrofit of structures to resist earthquake motions because it is designed to be computationally efficient for structures with a limited number of predefined nonlinear or energy dissipating elements. This is consistent with the modern philosophy of earthquake engineering that *energy dissipating elements should be able to be inspected and replaced after a major earthquake.*

Base isolators are one of the most common types of predefined nonlinear elements used in earthquake resistant designs. In addition, isolators, mechanical dampers, friction devices and plastic hinges are other types of common nonlinear elements. Also, gap elements are required to model contact between structural components and uplifting of structures. A special type of gap element with the ability to crush and dissipate energy is useful to model concrete and soil types of materials. Cables that can take tension only and dissipate energy in yielding are necessary to capture the behavior of many bridge type structures. In this chapter the behavior of several of those elements will be presented and detailed solution algorithms will be summarized.

21.2 GENERAL THREE-DIMENSIONAL TWO-NODE ELEMENT

The type of nonlinear element presented in this chapter is similar to the three-dimensional beam element. However, it can degenerate into an element with zero length where both ends are located at the same point in space. Therefore, it is possible to model sliding friction surfaces, contact problems and concentrated plastic hinges. Like the beam element, the user must define a local 1-2-3 reference system to define the local nonlinear element properties and to interpret the results. A typical element, connected between two points I and J, is shown in Figure 21.1.

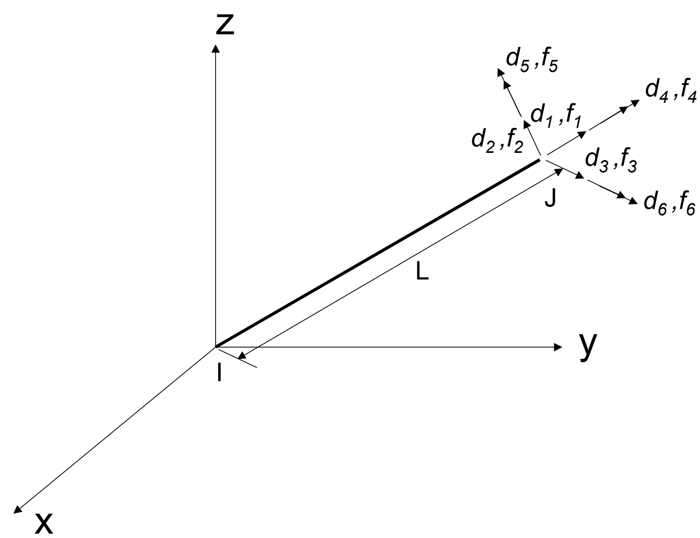

Figure 21.1 Relative Displacements - Three-Dimensional Nonlinear Element

It is important to note that three displacements and three rotations are possible at both points I and J and can be expressed in either the global X-Y-Z or local 1-2-3 reference system. The force and displacement transformation matrices for this nonlinear element are the same as for the beam element given in Chapter 4. For most element types, some of those displacements do not exist or are equal at I and J. Because each three-dimensional element has six rigid body displacements, the equilibrium of the element can be expressed in terms of the six relative displacements shown in Figure 21.1. Also, L can equal zero. For example, if a concentrated plastic hinge with a relative rotation about the local 2-axis is placed between points I and J, only a relative rotation d_5 exists. The other five relative displacements must be set to zero. This can be accomplished by setting the absolute displacements at joints I and J equal.

21.3 GENERAL PLASTICITY ELEMENT

{ XE "Plasticity Element" }The general plasticity element can be used to model many different types of nonlinear material properties. The fundamental properties and behavior of the element are illustrated in Figure 21.2.

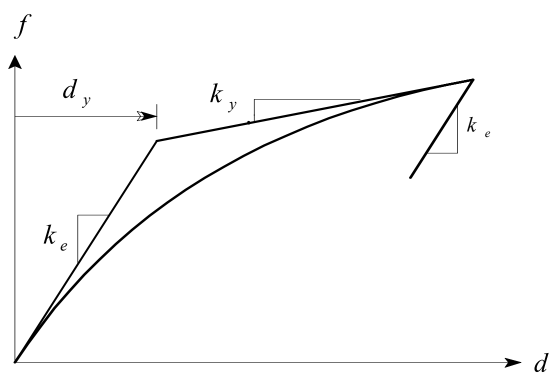

Figure 21.2 Fundamental Behavior of Plasticity Element

where k_e = initial linear stiffness

k_y = Yield stiffness

d_y = Yield deformation

The force-deformation relationship is calculated from:

$$f = k_y\, d + (k_e - k_y)\, e \tag{21.1}$$

Where d is the total deformation and e is an elastic deformation term and has a range $\pm d_y$. It is calculated at each time step by the numerical integration of one of the following differential equations:

$$\text{If} \quad \dot{d}e \geq 0 \quad \dot{e} = (1 - |\frac{e}{d_y}|^n)\dot{d} \tag{21.2}$$

$$\text{If} \quad \dot{d}e < 0 \quad \dot{e} = \dot{d} \tag{21.3}$$

The following finite difference approximations for each time step can be made:

$$\dot{d} = \frac{d_t - d_{t-\Delta t}}{\Delta t} \qquad \text{and} \qquad \dot{e} = \frac{e_t - e_{t-\Delta t}}{\Delta t} \tag{21.4a and 21.4b}$$

The numerical solution algorithm (six computer program statements) can be summarized at the end of each time increment Δt, at time t for iteration i, in Table 21.1.

Table 21.1 Iterative Algorithm for Plasticity Element

1. Change in deformation for time step Δt at time *t* for iteration *i*
$v = d_t^{(i)} - d_{t-\Delta t}$
2. Calculate elastic deformation for iteration *i*
if $v e_t^{(i-1)} \leq 0 \qquad e_t^{(i)} = e_{t-\Delta t} + v$
if $v e_t^{(i-1)} > 0 \qquad e_t^{(i)} = e_{t-\Delta t} + (1 - \lvert\frac{e_{t-\Delta t}}{d_y}\rvert^n)\, v$
if $e_t^{(i)} > d_y \qquad e_t^{(i)} = d_y$
if $e_t^{(i)} < -d_y \qquad e_t^{(i)} = -d_y$
3. Calculate iterative force:
$f_t^{(i)} = k_y d_t^{(i)} + (k_e - k_y) e_t^{(i)}$

Note that the approximate term $\frac{e_{t-\Delta t}}{d_y}$ is used from the end of the last time increment rather than the iterative term $\frac{e_t^{(i)}}{d_y}$. This approximation eliminates all problems associated with convergence for large values of n. However, the approximation has insignificant effects on the numerical results for all values of

n . For all practical purposes, a value of n equal to 20 produces true bilinear behavior.

21.4 DIFFERENT POSITIVE AND NEGATIVE PROPERTIES

{ XE "Algorithms for:Bilinear Plasticity Element" }The previously presented plasticity element can be generalized to have different positive, d_P , and negative, d_n , yield properties. This will allow the same element to model many different types of energy dissipation devices, such as the double diagonal Pall friction element.

Table 21.2 Iterative Algorithm for Non-Symmetric Bilinear Element

1. Change in deformation for time step Δt at time *t* for iteration *i*	
$v = d_t^{(i)} - d_{t-\Delta t}$	
2. Calculate elastic deformation for iteration *i*	
if $v e_t^{(i-1)} \le 0$	$e_t^{(i)} = e_{t-\Delta t} + v$
if $v e_t^{(i-1)} > 0$ and $e_{t-\Delta t} > 0$	$e_t^{(i)} = e_{t-\Delta t} + (1 - \lvert \frac{e_{t-\Delta t}}{d_p} \rvert^n) v$
if $v e_t^{(i-1)} > 0$ and $e_{t-\Delta t} < 0$	$e_t^{(i)} = e_{t-\Delta t} + (1 - \lvert \frac{e_{t-\Delta t}}{d_n} \rvert^n) v$
if $e_t^{(i)} > d_p$	$e_t^{(i)} = d_p$
if $e_t^{(i)} < -d_n$	$e_t^{(i)} = -d_n$
3. Calculate iterative force at time t :	
$f_t^{(i)} = k_y d_t^{(i)} + (k_e - k_y) e_t^{(i)}$	

For constant friction, the double diagonal Pall element has $k_e = 0$ and $n \approx 20$. For small forces both diagonals remain elastic, one in tension and one in compression. At some deformation, d_n, the compressive element may reach a maximum possible value. Friction slipping will start at the deformation d_p after which both the tension and compression forces will remain constant until the maximum displacement for the load cycle is obtained.

This element can be used to model bending hinges in beams or columns with non-symmetric sections. The numerical solution algorithm for the general bilinear plasticity element is given in Table 21.2.

21.5 THE BILINEAR TENSION-GAP-YIELD ELEMENT

{ XE "Tension-Gap-Yield Element" }The bilinear tension-only element can be used to model cables connected to different parts of the structure. In the retrofit of bridges, this type of element is often used at expansion joints to limit the relative movement during earthquake motions. The fundamental behavior of the element is summarized in Figure 21.3. The positive number d_0 is the axial deformation associated with initial cable sag. A negative number indicates an initial pre-stress deformation. The permanent element yield deformation is d_p.

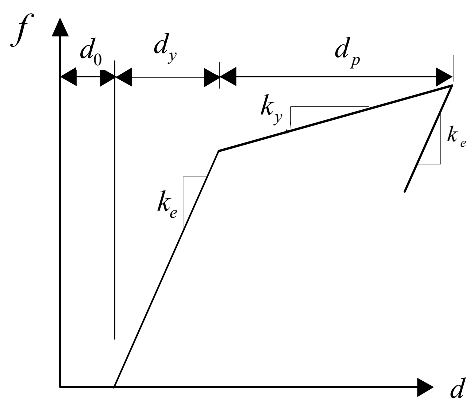

Figure 21.3 Tension-Gap-Yield Element

The numerical solution algorithm for this element is summarized in Table 21.3. Note that the permanent deformation calculation is based on the converged deformation at the end of the last time step. This avoids numerical solution problems.

{ XE "Algorithms for:Tension-Gap-Yield Element" }**Table 21.3 Iterative Algorithm for Tension-Gap-Yield Element**

1. Update Tension Yield Deformation from Previous Converged Time Step $y = d_{t-\Delta t} - d_0 - d_y$ if $y < d_p$ then $d_p = y$
2. Calculate Elastic Deformation for Iteration (*i*) $d = d_t^{(i)} - d_0$ $e_t^{(i)} = d - d_p$ if $e_t^{(i)} > d_y$ then $e_t^{(i)} = d_y$
3. Calculate Iterative Force: $f_t^{(i)} = k_y(d_t^{(i)} - d_0) + (k_e - k_y)e_t^{(i)}$ if $f_t^{(i)} < 0$ then $f_t^{(i)} = 0$

21.6 NONLINEAR GAP-CRUSH ELEMENT

Perhaps the most common type of nonlinear behavior that occurs in real structural systems is the closing of a gap between different parts of the structure; or, the uplifting of the structure at its foundation. The element can be used at abutment-soil interfaces and for modeling soil-pile contact. The gap/crush element has the following physical properties:

1. The element cannot develop a force until the opening d_0 gap is closed. A negative value of d_0 indicates an initial compression force.
2. The element can only develop a negative compression force. The first yield deformation d_y is specified by a positive number.
3. The crush deformation d_c is always a monotonically decreasing negative number.

The numerical algorithm for the gap-crush element is summarized in Table 21.4.

{ XE "Algorithms for:Gap-Crush Element" }**Table 21.4 Iterative Algorithm for Gap-Crush Element**

1. Update Crush Deformation from Previously Converged Time Step:
   $y = d_{t-\Delta t} + d_0 + d_y$
   if $y > d_c$ then $d_c = y$
2. Calculate Elastic Deformation:
   $e_t^{(i)} = d_t^{(i)} + d_o - d_c$
   if $e_t^{(i)} < -d_y$ then $e_t^{(i)} = -d_y$
3. Calculate Iterative Force:
   $f_t^{(i)} = k_y(d_t^{(i)} + d_0) + (k_e - k_y)e_t^{(i)}$
   if $f_t^{(i)} > 0$ then $f_t^{(i)} = 0$

The numerical convergence of the gap element can be very slow if a large elastic stiffness term k_e is used. The user must take great care in selecting a physically realistic number. To minimize numerical problems, the stiffness k_e should not be over 100 times the stiffness of the elements adjacent to the gap. The dynamic contact problem between real structural components often does not have a unique solution. Therefore, it is the responsibility of the design engineer to select materials at contact points and surfaces that have realistic material properties that can be predicted accurately.

21.7 VISCOUS DAMPING ELEMENTS

{ XE "Algorithms for:Damping Element" }{ XE "Algorithms for:Viscous Element" }Linear velocity-dependent energy-dissipation forces exist in only a few special materials subjected to small displacements. In terms of equivalent modal damping, experiments indicate that they are a small fraction of one percent. Manufactured mechanical dampers cannot be made with linear viscous properties because all fluids have finite compressibility and nonlinear behavior is present in all manmade devices. In the past it has been common practice to approximate the behavior of those viscous nonlinear elements by a simple linear viscous force. More recently, vendors of those devices have claimed that the damping forces are proportional to a power of the velocity. Experimental examination of a mechanical device indicates a far more complex behavior that cannot be represented by a simple one-element model.

The FNA method does not require that those damping devices be linearized or simplified to obtain a numerical solution. If the physical behavior is understood, it is possible for an iterative solution algorithm to be developed that will accurately simulate the behavior of almost any type of damping device. To illustrate the procedure, let us consider the device shown in Figure 21.4.

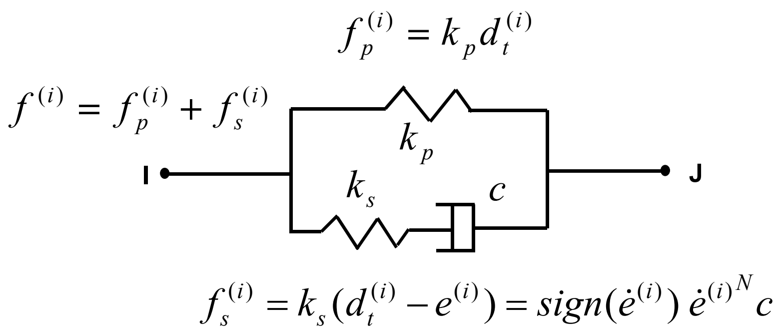

Figure 21.4 General Damping Element Connected Between Points I and J

It is apparent that the total deformation, $e_t^{(i)}$, across the damper must be accurately calculated to evaluate the equilibrium within the element at each time step. The finite difference equation used to estimate the damper deformation at time *t* is:

$$e_t^{(i)} = e_{t-\Delta t} + \int_{t-\Delta t}^{t} \dot{e}_\tau^{(i)} d\tau = e_{t-\Delta t} + \frac{\Delta t}{2}(\dot{e}_{t-\Delta t} + \dot{e}_t^{(i)}) \tag{21.5}$$

A summary of the numerical algorithm is summarized in Table 21.5.

{ XE "Algorithms for:Nonlinear Damping" }**Table 21.5 Iterative Algorithm for Nonlinear Viscous Element**

1. Estimate damper force from last iteration: $f_s^{(i)} = k_s(d_t^{(i)} - e_t^{(i-1)})$
2. Estimate damper velocity: $\dot{e}_t^{(i)} = (\frac{f_s^{(i)}}{c})^{\frac{1}{N}} sign(f_s^{(i)})$
3. Estimate damper deformation: $e_t^{(i)} = e_{t-\Delta t} + \frac{\Delta t}{2}(\dot{e}_{t-\Delta t)} + \dot{e}_t^{(i)})$
4. Calculate total iterative force: $f_t^{(i)} = k_p d_t^{(i)} + k_s(d_t^{(i)} - e_t^{(i)})$

21.8 THREE-DIMENSIONAL FRICTION-GAP ELEMENT

{ XE "Friction-Gap Element" }Many structures have contact surfaces between components of the structures or between structure and foundation that can only take compression. During the time the surfaces are in contact, it is possible for tangential friction forces to develop between the surfaces. The maximum tangential surface forces, which can be developed at a particular time, are a function of the normal compressive force that exists at that time. If the surfaces are not in contact, the normal and the surface friction forces must be zero. Therefore, surface slip displacements will take place during the period of time when the allowable friction force is exceeded or when the surfaces are not in contact.

To develop the numerical algorithm to predict the dynamic behavior between surfaces, consider the contact surface element shown in Figure 21.5. The two surface nodes are located at the same point in space and are connected by the gap-friction element that has contact stiffness k in all three directions. The three directions are defined by a local n, s and $s+90^o$ reference system. The element deformations d_n, d_s and d_{s+90} are relative to the absolute displacements of the two surfaces.

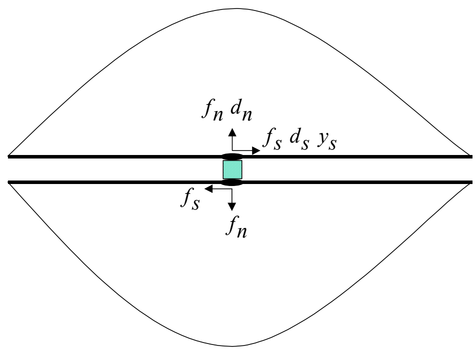

Figure 21.5 Three-Dimensional Nonlinear Friction-Gap Element

During the time of contact, the force-deformation relationships for the friction-gap element are:

Normal Force: $$f_n = kd_n \tag{21.6a}$$

Maximum Allowable Slip Force: $$f_a = \mu |f_n| \tag{21.6b}$$

Tangential Surface Forces: $$f_s = k(d_s - y_s) \quad \text{or,} \quad \bar{f}_s = sign(f_s) f_a \tag{21.6c}$$

The coefficient of sliding friction is designated by μ. The surface slip deformation in the s direction is y_s.

The iterative numerical algorithm for a typical time step is summarized in Table 21.6. To minimize numerical problems, the stiffness k should not be over 100 times the stiffness of the elements adjacent to the gap.

{ XE "Algorithms for:Friction-Gap Element" }**Table 21.6 Iterative Algorithm for Friction-Gap Element**

1. If $i=1$, update slip deformations from previously converged time step at s and $s+90^0$

   $$y_s(t) = y_s(t-\Delta t)$$

2. Evaluate normal and allowable slip forces

   if $d_n^{(i)} > 0 \quad f_n^{(i)} = 0$

   if $d_n^{(i)} \le 0 \quad f_n^{(i)} = kd_n^{(i)}$

   $f_a^{(i)} = \mu \left| f_n^{(i)} \right|$

3. Calculate surface forces at s and $s+90^0$

   if $d_n^{(i)} > 0 \quad f_s^{(i)} = 0$

   if $d_n^{(i)} \le 0 \quad f_s^{(i)} = k(d_s^{(i)} - y_s)$

   if $\left| f_s^{(i)} \right| > f_a^{(i)} \quad \bar{f}_s^{(i)} = sign(f_s^{(i)}) f_a^{(i)}$

4. Calculate slip deformations at s and $s+90^0$

   if $d_n^{(i)} > 0 \quad y_s^{(i)} = d_s^{(i)}$

   if $\left| f_s^{(i)} \right| = f_a^{(i)} \quad y_s^{(i)} = d_s^{(i)} - f_s^{(i)} / k$

21.9 SUMMARY

The use of approximate "equivalent linear viscous damping" has little theoretical or experimental justification and produces a mathematical model that violates dynamic equilibrium. It is now possible to accurately simulate the behavior of structures with a finite number of discrete gap, tension only, and energy dissipation devices installed. The experimentally determined properties of the devices can be directly incorporated into the computer model.