

# 16.

## SOIL STRUCTURE INTERACTION

*At a Finite Distance from a Structure,  
the Absolute Displacements  
Must Approach the Free-Field Displacements*

### 16.1 INTRODUCTION

{ XE "Soil-Structure Interactions" }The estimation of earthquake motions at the site of a structure is the most important phase of the design or retrofit of a structure. Because of the large number of assumptions required, experts in the field often disagree, by more than a factor of two, about the magnitude of motions expected at the site without the structure present. This lack of accuracy about the basic input motions, however, does not justify the introduction of additional unnecessary approximations in the dynamic analysis of the structure and its interaction with the material under the structure. Therefore, it will be assumed that the free-field motions at the location of the structure, without the structure present, can be estimated and are specified in the form of earthquake acceleration records in three directions. It is now common practice, on major engineering projects, to investigate several different sets of ground motions to consider both near fault and far fault events.

If a lightweight flexible structure is built on a very stiff rock foundation, a valid assumption is that the input motion at the base of the structure is the same as the free-field earthquake motion. This assumption is valid for a large number of building systems because most building type structures are approximately 90 percent voids, and it is not unusual for the weight of the structure to be equal to

the weight of the soil excavated before the structure is built. However, if the structure is very massive and stiff, such as a concrete gravity dam, and the foundation is relatively soft, the motion at the base of the structure may be significantly different from the free-field surface motion. Even for this extreme case, however, it is apparent that the most significant interaction effects will be near the structure, and, at some finite distance from the base of the structure, the displacements will converge back to the free-field earthquake motion.

## 16.2 SITE RESPONSE ANALYSIS

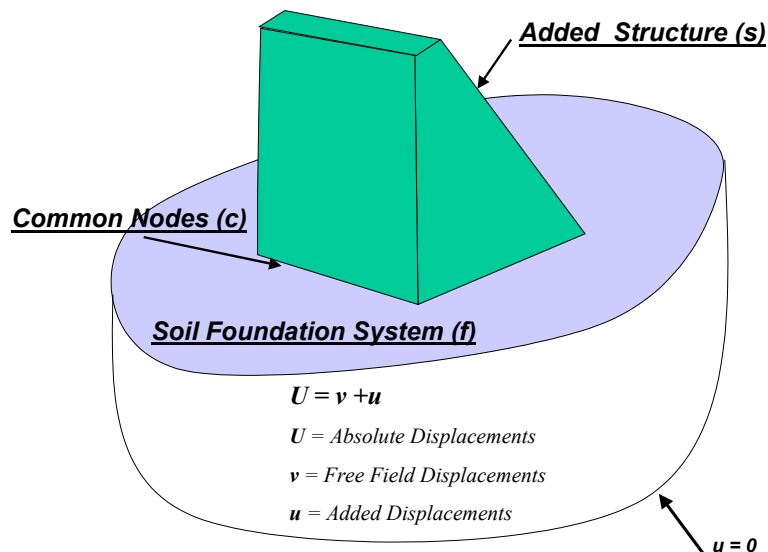
{ XE "Site Response Analysis" }The 1985 Mexico City and many other recent earthquakes clearly illustrate the importance of local soil properties on the earthquake response of structures. These earthquakes demonstrated that the rock motions could be amplified at the base of a structure by over a factor of five. Therefore, there is a strong engineering motivation for a site-dependent dynamic response analysis for many foundations to determine the free-field earthquake motions. The determination of a realistic site-dependent free-field surface motion at the base of a structure can be the most important step in the earthquake resistant design of any structure.

{ XE "SHAKE Program" }{ XE "WAVES Program" }For most horizontally layered sites, a one-dimensional pure shear model can be used to calculate the free-field surface displacements given the earthquake motion at the base of a soil deposit. Many special purpose computer programs exist for this purpose. SHAKE [1] is a well-known program that is based on the frequency domain solution method. SHAKE iterates to estimate effective linear stiffness and damping properties to approximate the nonlinear behavior of a site. WAVES [2] is a new nonlinear program in which the nonlinear equations of motion are solved using a direct step-by-step integration method. If the soil material can be considered linear, the SAP2000 program, using the SOLID element, can calculate either the one-, two- or three-dimensional free-field motions at the base of a structure. In addition, a one-dimensional nonlinear site analysis can be accurately conducted using the FNA option in the SAP2000 program.

### 16.3 KINEMATIC OR SOIL STRUCTURE INTERACTION

The most common soil structure interaction (SSI) approach used for three-dimensional soil structure systems is based on the "added motion" formulation [3]. This formulation is mathematically simple, theoretically correct, and is easy to automate and use within a general linear structural analysis program. In addition, the formulation is valid for free-field motions caused by earthquake waves generated from all sources. The method requires that the free-field motions at the base of the structure be calculated before the soil structure interactive analysis.

To develop the fundamental SSI dynamic equilibrium equations, consider the three-dimensional soil structure system shown in Figure 16.1.



**Figure 16.1 Soil structure Interaction Model**

Consider the case where the SSI model is divided into three sets of node points. The common nodes at the interface of the structure and foundation are identified with "c"; the other nodes within the structure are "s" nodes; and the other nodes within the foundation are "f" nodes. From the direct stiffness approach in

structural analysis, the dynamic force equilibrium of the system is given in terms of the **absolute displacements**,  $\mathbf{U}$ , by the following sub-matrix equation:

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{U}}_c \\ \ddot{\mathbf{U}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sf} & \mathbf{0} \\ \mathbf{K}_{cf} & \mathbf{K}_{cc} & \mathbf{K}_{cf} \\ \mathbf{0} & \mathbf{K}_{fc} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{U}_s \\ \mathbf{U}_c \\ \mathbf{U}_f \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (16.1)$$

where the mass and the stiffness at the contact nodes are the sum of the contributions from the structure ( $s$ ) and foundation ( $f$ ), and are given by:

$$\mathbf{M}_{cc} = \mathbf{M}_{cc}^{(s)} + \mathbf{M}_{cc}^{(f)} \quad \text{and} \quad \mathbf{K}_{cc} = \mathbf{K}_{cc}^{(s)} + \mathbf{K}_{cc}^{(f)} \quad (16.2)$$

{ XE "Free-Field Displacements" } In terms of absolute motion, there are no external forces acting on the system. However, the displacements at the boundary of the foundation must be known. To avoid solving this SSI problem directly, the dynamic response of the foundation without the structure is calculated. In many cases, this **free-field** solution can be obtained from a simple one-dimensional site model. The three-dimensional free-field solution is designated by the absolute displacements  $\mathbf{v}$  and absolute accelerations  $\ddot{\mathbf{v}}$ . By a simple change of variables, it is now possible to express the absolute displacements  $\mathbf{U}$  and accelerations  $\ddot{\mathbf{U}}$  in terms of displacements  $\mathbf{u}$  relative to the free-field displacements  $\mathbf{v}$ . Or:

$$\begin{bmatrix} \mathbf{U}_s \\ \mathbf{U}_c \\ \mathbf{U}_f \end{bmatrix} \equiv \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_c \\ \mathbf{u}_f \end{bmatrix} + \begin{bmatrix} \mathbf{v}_s \\ \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{U}}_c \\ \ddot{\mathbf{U}}_f \end{bmatrix} \equiv \begin{bmatrix} \ddot{\mathbf{u}}_s \\ \ddot{\mathbf{u}}_c \\ \ddot{\mathbf{u}}_f \end{bmatrix} + \begin{bmatrix} \ddot{\mathbf{v}}_s \\ \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} \quad (16.3)$$

Equation (16.1) can now be written as

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_s \\ \ddot{\mathbf{u}}_c \\ \ddot{\mathbf{u}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sc} & \mathbf{0} \\ \mathbf{K}_{cs} & \mathbf{K}_{cc} & \mathbf{K}_{cf} \\ \mathbf{0} & \mathbf{K}_{fc} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_c \\ \mathbf{u}_f \end{bmatrix} = \quad (16.4)$$

$$- \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}}_s \\ \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} - \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sc} & \mathbf{0} \\ \mathbf{K}_{cs} & \mathbf{K}_{cc} & \mathbf{K}_{cf} \\ \mathbf{0} & \mathbf{K}_{fc} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} = \mathbf{R}$$

If the free-field displacement  $\mathbf{v}_c$  is constant over the base of the structure, the term  $\mathbf{v}_s$  is the rigid body motion of the structure. Therefore, Equation (16.4) can be further simplified by the fact that the static rigid body motion of the structure is:

$$\begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sc} \\ \mathbf{K}_{cs} & \mathbf{K}_{cc}^{(s)} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{v}_c \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (16.5)$$

Also, the dynamic free-field motion of the foundation requires that:

$$\begin{bmatrix} \mathbf{M}_{cc}^{(f)} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{cc}^{(f)} & \mathbf{K}_{cf} \\ \mathbf{K}_{cf} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (16.6)$$

Therefore, the right-hand side of Equation (16.4) can be written as:

$$\mathbf{R} = \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc}^{(s)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}}_s \\ \ddot{\mathbf{v}}_c \\ \mathbf{0} \end{bmatrix} \quad (16.7)$$

Hence, the right-hand side of the Equation (16.4) does not contain the mass of the foundation. Therefore, three-dimensional dynamic equilibrium equations for the complete soil structure system with damping added are of the following form for a lumped mass system:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{m}_x \ddot{v}_x(t) - \mathbf{m}_y \ddot{v}_y(t) - \mathbf{m}_z \ddot{v}_z(t) \quad (16.8)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping and stiffness matrices, respectively, of

the soil structure model. The added relative displacements,  $\mathbf{u}$ , exist for the soil structure system and must be set to zero at the sides and bottom of the foundation. The terms  $\ddot{v}_x(t)$ ,  $\ddot{v}_y(t)$  and  $\ddot{v}_z(t)$  are the free-field components of the acceleration if the structure is not present. The column matrices,  $\mathbf{m}_i$ , are the directional masses for the added structure only.

Most structural analysis computer programs automatically apply the seismic loading to all mass degrees of freedom within the computer model and cannot solve the SSI problem. This lack of capability has motivated the development of the massless foundation model. This allows the correct seismic forces to be applied to the structure; however, the inertia forces within the foundation material are neglected. The results from a massless foundation analysis converge as the size of the foundation model is increased. However, the converged solutions may have avoidable errors in the mode shapes, frequencies and response of the system.

To activate the soil structure interaction within a computer program, it is only necessary to identify the foundation mass so that the loading is not applied to that part of the structure. The program then has the required information to form both the total mass and the mass of the added structure. The SAP2000 program has this option and is capable of solving the SSI problem correctly.

#### 16.4 RESPONSE DUE TO MULTI-SUPPORT INPUT MOTIONS

{ XE "Multi-Support Earthquake Motions" } The previous SSI analysis assumes that the free-field motion at the base of the structure is constant. For large structures such as bridges and arch dams, the free-field motion is not constant at all points where the structure is in contact with the foundation.

The approach normally used to solve this problem is to define a *quasi-static displacement*  $v_c$  that is calculated from the following equation:

$$\mathbf{K}_{ss}\mathbf{v}_s + \mathbf{K}_{sc}\mathbf{v}_c = \mathbf{0} \quad \text{or,} \quad \mathbf{v}_s = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sc}\mathbf{v}_c = \mathbf{T}_{sc}\mathbf{v}_c \quad (16.9a)$$

The transformation matrix  $\mathbf{T}_{sc}$  allows the corresponding quasi-static acceleration in the structure to be calculated from:

$$\ddot{\mathbf{v}}_s = \mathbf{T}_{sc} \ddot{\mathbf{v}}_c \quad (16.9b)$$

Equation (16.4) can be written as:

$$\mathbf{R} = - \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}}_s \\ \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} - \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sc} & \mathbf{0} \\ \mathbf{K}_{cs} & \mathbf{K}_{cc} & \mathbf{K}_{cf} \\ \mathbf{0} & \mathbf{K}_{fc} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} \quad (16.10)$$

After substitution of Equations (16.6) and (16.9), Equation (16.10) can be written as:

$$\mathbf{R} = - \begin{bmatrix} \mathbf{0} & \mathbf{M}_{ss} \mathbf{T}_{sc} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}}_s \\ \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{K}}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} \quad (16.11)$$

The reduced structural stiffness at the contact surface  $\overline{\mathbf{K}}_{cc}$  is given by:

$$\overline{\mathbf{K}}_{cc} = \mathbf{K}_{cc} + \mathbf{K}_{cs} \mathbf{T}_{sc} \quad (16.12)$$

Therefore, this approach requires a special program option to calculate the mass and stiffness matrices to be used on the right-hand side of the dynamic equilibrium equations. Note that the loads are a function of both the free-field displacements and accelerations at the soil structure contact. Also, to obtain the total stresses and displacements within the structure, the quasi-static solution must be added to the solution. At the present time, no general purpose structural analysis computer program is based on this “numerically cumbersome” approach.

An alternative approach is to formulate the solution directly in terms of the absolute displacements of the structure. This involves the introduction of the following change of variables:

$$\begin{bmatrix} \mathbf{U}_s \\ \mathbf{U}_c \\ \mathbf{U}_f \end{bmatrix} \equiv \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_c \\ \mathbf{u}_f \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{v}_c \\ \mathbf{v}_f \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{U}}_c \\ \ddot{\mathbf{U}}_f \end{bmatrix} \equiv \begin{bmatrix} \ddot{\mathbf{u}}_s \\ \ddot{\mathbf{u}}_c \\ \ddot{\mathbf{u}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \ddot{\mathbf{v}}_c \\ \ddot{\mathbf{v}}_f \end{bmatrix} \quad (16.13)$$

Substitution of this change of variables into Equation (16.1) yields the following dynamic equilibrium equations in terms of the absolute displacement,  $\mathbf{u}_s$ , of the structure:

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_s \\ \ddot{\mathbf{u}}_c \\ \ddot{\mathbf{u}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sf} & \mathbf{0} \\ \mathbf{K}_{cf} & \mathbf{K}_{cc} & \mathbf{K}_{cf} \\ \mathbf{0} & \mathbf{K}_{fc} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_c \\ \mathbf{u}_f \end{bmatrix} = \mathbf{R} \quad (16.14)$$

After the free-field response, Equation (16.6), has been removed, the dynamic loading is calculated from the following equation:

$$\mathbf{R} = - \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sc} & \mathbf{0} \\ \mathbf{K}_{cs} & \mathbf{K}_{cc}^{(s)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{v}_c \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{cc}^{(s)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \dot{\mathbf{v}}_c \\ \mathbf{0} \end{bmatrix} \quad (16.15a)$$

This equation can be further simplified by connecting the structure to the foundation with stiff massless springs that are considered as part of the structure. Therefore, the mass of the structure at the contact nodes is eliminated and Equation (16.15a) is reduced to:

$$\mathbf{R} = - \begin{bmatrix} \mathbf{K}_{sc} \\ \mathbf{K}_{cc}^{(s)} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \end{bmatrix} \quad (16.15b)$$

It is apparent that the stiffness terms in Equation (16.15b) represent the stiffness of the contact springs only. Therefore, for a typical displacement component ( $n = x, y$  or  $z$ ), the forces acting at point “i” on the structure and point “j” on the foundation are given by:

$$\begin{bmatrix} R_i \\ R_j \end{bmatrix}_n = -k_n \begin{bmatrix} 1.0 & -1.0 \\ -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{v}_n \end{bmatrix} \quad (16.16)$$

where  $k_n$  is the massless spring stiffness in the  $n$ th direction and  $\mathbf{v}_n$  is the free-field displacement. Hence, points “i” and “j” can be at the same location in space and the only loads acting are a series of time-dependent, concentrated point loads

that are equal and opposite forces between the structure and foundation. The spring stiffness selected must be approximately three orders-of-magnitude greater than the stiffness of the structure at the connecting nodes. The spring stiffness should be large enough so the fundamental periods of the system are not changed, and small enough not to cause numerical problems.

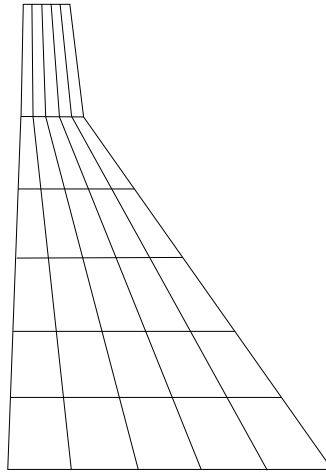
The dynamic equilibrium equations, with damping added, can be written in the following form:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{R} \quad (16.17)$$

{ XE "LDR Vectors" } It should be noted that concentrated dynamic loads generally require a large number of eigenvectors to capture the correct response of the system. However, if LDR vectors are used in a mode superposition analysis, the required number of vectors is reduced significantly. The SAP2000 program has the ability to solve the multi-support, soil structure interaction problems using this approach. At the same time, selective nonlinear behavior of the structure can be considered.

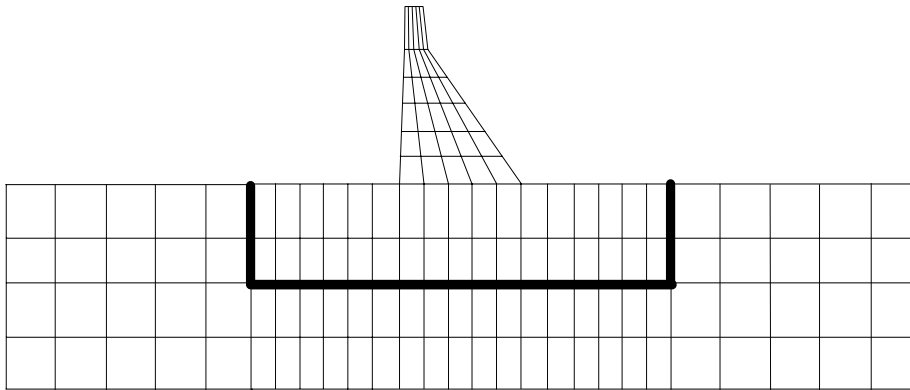
## 16.5 ANALYSIS OF GRAVITY DAM AND FOUNDATION

To illustrate the use of the soil structure interaction option, several earthquake response analyses of the Pine Flat Dam were conducted using different foundation models. The foundation properties were assumed to be the same properties as the dam. Damping was set at five percent. Ten Ritz vectors generated from loads on the dam only were used. However, the resulting approximate mode shapes used in the standard mode superposition analysis included the mass inertia effects of the foundation. The horizontal dynamic loading was the typical segment of the Loma Prieta earthquake defined in Figure 15.1a. A finite element model of the dam on a rigid foundation is shown in Figure 16.2.



**Figure 16.2** *Finite Element Model of Dam Only*

The two different foundation models used are shown in Figure 16.3.



**Figure 16.3** *Models of Dam with Small and Large Foundation*

Selective results are summarized in Table 16.1. For the purpose of comparison, it will be assumed that Ritz vector results for the large foundation mesh are the referenced values.

**Table 16.1 Selective Results of Dam-Foundation Analyses**

Dam Foundation	Total Mass (lb-sec <sup>2</sup> /in)	Periods (seconds)	Max. Displacement (inches)	Max. & Min. Stress (ksi)
None	1,870	0.335 0.158	0.65	-37 to +383
Small	13,250	0.404 0.210	1.28	-490 to +289
Large	77,360	0.455 0.371	1.31	-512 to +297

The differences between the results of the small and large foundation models are very close, which indicates that the solution of the large foundation model may be nearly converged. It is true that the radiation damping effects in a finite foundation model are neglected. However, as the foundation model becomes larger, the energy dissipation from normal modal damping within the massive foundation is significantly larger than the effects of radiation damping for transient earthquake-type loading.

## 16.6 THE MASSLESS FOUNDATION APPROXIMATION

{ XE "Massless Foundation Approximation" } Most general purpose programs for earthquake analysis of structures do not have the option of identifying the foundation mass as a separate type of mass on which the earthquake forces do not act. Therefore, an approximation that has commonly been used is to neglect the mass of the foundation completely in the analysis. Table 16.2 summarizes the results for an analysis of the same dam-foundation systems using a massless foundation. As expected, these results are similar. For this case the results are conservative; however, one cannot be assured of this for all cases.

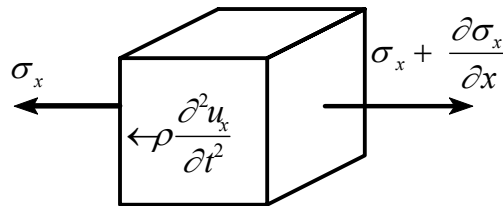
**Table 16.2 Selective Results of Dam With Massless Foundation Analyses**

Dam Foundation	Total Mass (lb-sec <sup>2</sup> /in)	Periods (seconds)	Max. Displacement (inches)	Max. & Min. Stress (ksi)
None	1,870	0.335 0.158	0.65	-37 to +383
Small	1,870	0.400 0.195	1.27	-480 to +289
Large	1,870	0.415 0.207	1.43	-550 to +330

## 16.7 APPROXIMATE RADIATION BOUNDARY CONDITIONS

{ XE "Radiation Boundary Conditions" } If the foundation volume is large and the modal damping exists, it was demonstrated in the previous section that a finite foundation with fixed boundaries can produce converged results. However, the use of energy absorbing boundaries can further reduce the size of the foundation required to produce a converged solution.

{ XE "Wave Propagation" } To calculate the properties of this boundary condition, consider a plane wave propagating in the x-direction. The forces that cause wave propagation are shown acting on a unit cube in Figure 16.4.



*Figure 16.4 Forces Acting on Unit Cube*

From Figure 16.4 the one dimensional equilibrium equation in the x-direction is:

$$\rho \frac{\partial^2 u}{\partial t^2} - \frac{\partial \sigma_x}{\partial x} = 0 \quad (16.18)$$

Because  $\sigma_x = \lambda \varepsilon_x = \lambda \frac{\partial u}{\partial x}$ , the one-dimensional partial differential equation is written in the following classical wave propagation form:

$$\frac{\partial^2 u}{\partial t^2} - V_p^2 \frac{\partial^2 u_x}{\partial x^2} = 0 \quad (16.19)$$

where  $V_p$  is the wave propagation velocity of the material and is given by:

$$V_p = \sqrt{\frac{\lambda}{\rho}} \quad (16.20)$$

in which  $\rho$  is the mass density and  $\lambda$  is the bulk modulus given by:

$$\lambda = \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} E \quad (16.21)$$

The solution of Equation (16.13) for harmonic wave propagation in the positive x-direction is a displacement of the following form:

$$u(t, x) = U \left[ \sin\left(\omega t - \frac{\omega x}{V_p}\right) + \cos\left(\omega t - \frac{\omega x}{V_p}\right) \right] \quad (16.22)$$

This equation can be easily verified by substitution into Equation (16.18). The arbitrary frequency of the harmonic motion is  $\omega$ . The velocity,  $\frac{\partial u}{\partial t}$ , of a particle at location x is:

$$\dot{u}(t, x) = U\omega \left[ \cos\left(\omega t - \frac{\omega x}{V_p}\right) - \sin\left(\omega t - \frac{\omega x}{V_p}\right) \right] \quad (16.23)$$

The strain in the x-direction is:

$$\varepsilon(x, t) = \frac{\partial u}{\partial x} = -\frac{\dot{u}(x, t)}{V_p} \quad (16.24)$$

The corresponding stress can now be expressed in the following simplified form:

$$\sigma(x, t) = \lambda \varepsilon(x, t) = -V_p \rho \dot{u}(x, t) \quad (16.25)$$

The compression stress is identical to the force in a simple viscous damper with constant damping value equal to  $V_p \rho$  per unit area of the boundary. Therefore, a boundary condition can be created, at a cut boundary, which will allow the wave to pass without reflection and allow the strain energy to “radiate” away from the foundation.

Also, it can be easily shown that the shear wave “radiation” boundary condition, parallel to a free boundary, is satisfied if damping values are assigned to be  $V_s \rho$  per unit of boundary area. The shear wave propagation velocity is given by:

$$V_s = \sqrt{\frac{G}{\rho}} \quad (16.26)$$

where  $G$  is the shear modulus.

The FNA method can be used to solve structures in the time domain with these types of boundary conditions. In later editions of this book, the accuracy of those boundary condition approximations will be illustrated using numerical examples. Also, it will be used with a fluid boundary where only compression waves exist.

## 16.8 USE OF SPRINGS AT THE BASE OF A STRUCTURE

{ XE "Half-Space Equations" } Another important structural modeling problem that must be solved is at the interface of the major structural elements within a structure and the foundation material. For example, the deformations at the base of a major shear wall in a building structure will significantly affect the displacement and force distribution in the upper stories of a building for both static and dynamic loads. Realistic spring stiffness can be selected from separate finite element studies or by using the classical half-space equations given in Table 16.3.

It is the opinion of the author that the use of appropriate site-dependent free-field earthquake motions and selection of realistic massless springs at the base of the structure are the only modeling assumptions required to include site and

foundation properties in the earthquake analysis of most structural systems.

Table 16.3 also contains effective mass and damping factors that include the approximate effects of radiation damping. Those values can be used directly in a computer model without any difficulty. However, considerable care should be taken in using those equations at the base of a complete structure. For example, the effective earthquake forces must not be applied to the foundation mass.

**Table 16.3 Properties of Rigid Circular Plate on Surface of Half-Space**

DIRECTION	STIFFNESS	DAMPING	MASS
Vertical	$K = \frac{4Gr}{1-\nu}$	$1.79\sqrt{K\rho}r^3$	$1.50\rho r^3$
Horizontal	$18.2Gr \frac{(1-\nu^2)}{(2-\nu)^2}$	$1.08\sqrt{K\rho}r^3$	$0.28\rho r^3$
Rotation	$2.7Gr^3$	$0.47\sqrt{K\rho}r^3$	$0.49\rho r^5$
Torsion	$5.3Gr^3$	$1.11\sqrt{K\rho}r^5$	$0.70\rho r^5$

$r$  = plate radius;  $G$  = shear modulus;  $\nu$  = Poisson's ratio;  $\rho$  = mass density

Source: Adapted from "Fundamentals of Earthquake Engineering, by Newmark and Rosenblueth, Prentice-Hall, 1971.

## 16.9 SUMMARY

A large number of research papers and several books have been written on structure-foundation-soil analysis and site response from earthquake loading. However, the majority of those publications have been restricted to the linear behavior of soil structure systems. It is possible to use the numerical methods presented in this book to conduct accurate earthquake analysis of real soil structure systems in the time domain, including many realistic nonlinear properties. Also, it can be demonstrated that the solution obtained is converged to the correct soil structure interactive solution.

For major structures on soft soil, one-dimensional site response analyses should

be conducted. Under major structural elements, such as the base of a shear wall, massless elastic springs should be used to estimate the foundation stiffness. For massive structures, such as gravity dams, a part of the foundation should be modeled using three-dimensional SOLID elements in which SSI effects are included.

## 16.10 REFERENCES

1. { XE "Lysmer, J." } Schnabel, P., J. Lysmer and H. Seed. 1970. "SHAKE - A Computer Program for the Earthquake Response for Horizontally Layered Sites," EERC Report No. 72-2. University of California, Berkeley. February.
2. { XE "Hart, J." } { XE "WAVES Program" } Hart, J., and E. Wilson. "WAVES - An Efficient Microcomputer Program for Nonlinear Site Response Analysis," National Information Center for Earthquake Engineering. Davis Hall, University of California, Berkeley. Tel. # (415) 642-5113.
3. { XE "Clough, Ray W." } { XE "Penzien, J." } Clough, R., and J. Penzien. 1993. *Dynamics of Structures*, Second Edition. McGraw-Hill, Inc. ISBN 0-07-011394-7.