12.

DYNAMIC ANALYSIS

Force Equilibrium is Fundamental in the Dynamic Analysis of Structures

12.1 INTRODUCTION

{ XE "Newton's Second Law" }All real physical structures behave dynamically when subjected to loads or displacements. The additional inertia forces, ***from Newton's second law***, are equal to the mass times the acceleration. If the loads or displacements are applied very slowly, the inertia forces can be neglected and a static load analysis can be justified. Hence, dynamic analysis is a simple extension of static analysis.

In addition, all real structures potentially have an infinite number of displacements. Therefore, the most critical phase of a structural analysis is to create a computer model with a finite number of massless members and a finite number of node (joint) displacements that will simulate the behavior of the real structure. The mass of a structural system, which can be accurately estimated, is lumped at the nodes. Also, for linear elastic structures, the stiffness properties of the members can be approximated with a high degree of confidence with the aid of experimental data. However, the dynamic loading, energy dissipation properties and boundary (foundation) conditions for many structures are difficult to estimate. This is always true for the cases of seismic input or wind loads.

To reduce the errors that may be caused by the approximations summarized in the previous paragraph, it is necessary to conduct many different dynamic analyses using different computer models, loading and boundary conditions. It is not unrealistic to conduct 20 or more computer runs to design a new structure or to investigate retrofit options for an existing structure.

Because of the large number of computer runs required for a typical dynamic analysis, it is very important that accurate and numerically efficient methods be used within computer programs. Some of those methods have been developed by the author and are relatively new. Therefore, one of the purposes of this book is to summarize those numerical algorithms, their advantages and limitations.

12.2 DYNAMIC EQUILIBRIUM

The force equilibrium of a multi-degree-of-freedom lumped mass system as a function of time can be expressed by the following relationship:

$$\mathbf{F}(t)_I + \mathbf{F}(t)_D + \mathbf{F}(t)_S = \mathbf{F}(t) \tag{12.1}$$

in which the force vectors at time t are:

$\mathbf{F}(t)_I$ is a vector of inertia forces acting on the node masses

$\mathbf{F}(t)_D$ is a vector of viscous damping, or energy dissipation, forces

$\mathbf{F}(t)_S$ is a vector of internal forces carried by the structure

$\mathbf{F}(t)$ is a vector of externally applied loads

Equation (12.1) is based on physical laws and is valid for both linear and nonlinear systems if equilibrium is formulated with respect to the deformed geometry of the structure.

For many structural systems, the approximation of linear structural behavior is made to convert the physical equilibrium statement, Equation (12.1), to the following set of second-order, linear, differential equations:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t)_a + \mathbf{C}\dot{\mathbf{u}}(t)_a + \mathbf{K}\mathbf{u}(t)_a = \mathbf{F}(t) \tag{12.2}$$

in which $\mathbf{M}$ is the mass matrix (lumped or consistent), $\mathbf{C}$ is a viscous damping matrix (which is normally selected to approximate energy dissipation in the real structure) and $\mathbf{K}$ is the static stiffness matrix for the system of structural elements. The time-dependent vectors $\mathbf{u}(t)_a$, $\dot{\mathbf{u}}(t)_a$ and $\ddot{\mathbf{u}}(t)_a$ are the absolute node displacements, velocities and accelerations, respectively.

Many books on structural dynamics present several different methods of applied mathematics to obtain the exact solution of Equation (12.2). Within the past several years, however, with the general availability of inexpensive, high-speed personal computers (see Appendix H), the exact solution of Equation (12.2) can be obtained without the use of complex mathematical techniques. Therefore, the modern structural engineer who has a physical understanding of dynamic equilibrium and energy dissipation can perform dynamic analysis of complex structural systems. A strong engineering mathematical background is desirable; however, in my opinion, it is no longer mandatory.

{ XE "Earthquake Loading" }For seismic loading, the external loading $\mathbf{F}(t)$ is equal to zero. The basic seismic motions are the three components of free-field ground displacements $u(t)_{ig}$ that are known at some point below the foundation level of the structure. Therefore, we can write Equation (12.2) in terms of the displacements $\mathbf{u}(t)$, velocities $\dot{\mathbf{u}}(t)$ and accelerations $\ddot{\mathbf{u}}(t)$ that are relative to the three components of free-field ground displacements.

Therefore, the absolute displacements, velocities and accelerations can be eliminated from Equation (12.2) by writing the following simple equations:

$$\begin{aligned}
\mathbf{u}(t)_a &= \mathbf{u}(t) + \mathbf{I}_x u(t)_{xg} + \mathbf{I}_y u(t)_{yg} + \mathbf{I}_z u(t)_{zg} \\
\dot{\mathbf{u}}(t)_a &= \dot{\mathbf{u}}(t) + \mathbf{I}_x \dot{u}(t)_{xg} + \mathbf{I}_y \dot{u}(t)_{yg} + \mathbf{I}_z \dot{u}(t)_{zg} \\
\ddot{\mathbf{u}}(t)_a &= \ddot{\mathbf{u}}(t) + \mathbf{I}_x \ddot{u}(t)_{xg} + \mathbf{I}_y \ddot{u}(t)_{yg} + \mathbf{I}_z \ddot{u}(t)_{zg}
\end{aligned} \tag{12.3}$$

where $\mathbf{I}_i$ is a vector with ones in the "i" directional degrees-of-freedom and zero in all other positions. The substitution of Equation (12.3) into Equation (12.2) allows the node point equilibrium equations to be rewritten as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}_x \ddot{u}(t)_{xg} - \mathbf{M}_y \ddot{u}(t)_{yg} - \mathbf{M}_z \ddot{u}(t)_{zg} \tag{12.4}$$

where $\mathbf{M}_i = \mathbf{M}\mathbf{I}_i$.

The simplified form of Equation (12.4) is possible since the rigid body velocities and displacements associated with the base motions cause no additional damping or structural forces to be developed.

It is important for engineers to realize that the displacements, which are normally printed by a computer program, are relative displacements and that the fundamental loading on the structure is foundation displacements and not externally applied loads at the joints of the structure. For example, the static pushover analysis of a structure is a poor approximation of the dynamic behavior of a three-dimensional structure subjected to complex time-dependent base motions. Also, one must calculate absolute displacements to properly evaluate base isolation systems.

There are several different classical methods that can be used for the solution of Equation (12.4). Each method has advantages and disadvantages that depend on the type of structure and loading. To provide a general background for the various topics presented in this book, the different numerical solution methods are summarized below.

12.3 STEP-BY-STEP SOLUTION METHOD

{ XE "Dynamic Analysis by:Direct Integration" }The most general solution method for dynamic analysis is an incremental method in which the equilibrium equations are solved at times $\Delta t, 2\Delta t, 3\Delta t$, etc. There are a large number of different incremental solution methods. In general, they involve a solution of the complete set of equilibrium equations at each time increment. In the case of nonlinear analysis, it may be necessary to reform the stiffness matrix for the complete structural system for each time step. Also, iteration may be required within each time increment to satisfy equilibrium. As a result of the large computational requirements, it can take a significant amount of time to solve structural systems with just a few hundred degrees-of-freedom.

In addition, artificial or numerical damping must be added to most incremental solution methods to obtain stable solutions. For this reason, engineers must be very careful in the interpretation of the results. For some nonlinear structures subjected to seismic motions, incremental solution methods are necessary.

For very large structural systems, a combination of mode superposition and incremental methods has been found to be efficient for systems with a small number of nonlinear members. This method has been incorporated into the new versions of SAP and ETABS and will be presented in detail later in this book.

12.4 MODE SUPERPOSITION METHOD

{ XE "Dynamic Analysis by:Mode Superposition" }The most common and effective approach for seismic analysis of linear structural systems is the mode superposition method. After a set of orthogonal vectors have been evaluated, this method reduces the large set of global equilibrium equations to a relatively small number of uncoupled second order differential equations. The numerical solution of those equations involves greatly reduced computational time.

It has been shown that seismic motions excite only the lower frequencies of the structure. Typically, earthquake ground accelerations are recorded at increments of 200 points per second. Therefore, the basic loading data does not contain information over 50 cycles per second. Hence, neglecting the higher frequencies and mode shapes of the system normally does not introduce errors.

12.5 RESPONSE SPECTRA ANALYSIS

{ XE "Dynamic Analysis by:Response Spectrum" }The basic mode superposition method, which is restricted to linearly elastic analysis, produces the complete time history response of joint displacements and member forces because of a specific ground motion loading [1, 2]. There are two major disadvantages of using this approach. First, the method produces a large amount of output information that can require an enormous amount of computational effort to conduct all possible design checks as a function of time. Second, the analysis must be repeated for several different earthquake motions to ensure that all the significant modes are excited, because a response spectrum for one earthquake, in a specified direction, is not a smooth function.

There are significant computational advantages in using the response spectra method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode using smooth design spectra that are the average of several earthquake motions. In this book, we will recommend the CQC method to combine these maximum modal response values to obtain the most probable peak value of displacement or force. In addition, it will be shown that the SRSS and CQC3 methods of combining results from orthogonal earthquake motions will allow one dynamic analysis to produce design forces for all members in the structure.

12.6 SOLUTION IN THE FREQUENCY DOMAIN

{ XE "Dynamic Analysis by:Frequency Domain" }The basic approach used to solve the dynamic equilibrium equations in the frequency domain is to expand the external loads $\mathbf{F}(t)$ in terms of Fourier series or Fourier integrals. The solution is in terms of complex numbers that cover the time span from $-\infty$ to ∞. Therefore, it is very effective for periodic types of loads such as mechanical vibrations, acoustics, sea-waves and wind [1]. However, the use of the frequency domain solution method for solving structures subjected to earthquake motions has the following disadvantages:

1. The mathematics for most structural engineers, including myself, is difficult to understand. Also, the solutions are difficult to verify.

2. Earthquake loading is not periodic; therefore, it is necessary to select a long time period so that the solution from a finite length earthquake is completely damped out before application of the same earthquake at the start of the next period of loading.

3. For seismic type loading, the method is not numerically efficient. The transformation of the result from the frequency domain to the time domain, even with the use of Fast Fourier Transformation methods, requires a significant amount of computational effort.

4. The method is restricted to the solution of linear structural systems.

5. The method has been used, without sufficient theoretical justification, for the approximate nonlinear solution of site response problems and soil/structure

interaction problems. Typically, it is used in an iterative manner to create linear equations. The linear damping terms are changed after each iteration to approximate the energy dissipation in the soil. Hence, dynamic equilibrium within the soil is not satisfied.

12.7 SOLUTION OF LINEAR EQUATIONS

{ XE "Solution of Equations" }The step-by-step solution of the dynamic equilibrium equations, the solution in the frequency domain, and the evaluation of eigenvectors and Ritz vectors all require the solution of linear equations of the following form:

$$\mathbf{AX} = \mathbf{B} \tag{12.5}$$

Where **A** is an 'N by N' symmetric matrix that contains a large number of zero terms. The 'N by M' **X** displacement and **B** load matrices indicate that more than one load condition can be solved at the same time.

The method used in many computer programs, including SAP2000 [5] and ETABS [6], is based on the profile or active column method of compact storage. Because the matrix is symmetric, it is only necessary to form and store the first non-zero term in each column down to the diagonal term in that column. Therefore, the sparse square matrix can be stored as a one-dimensional array along with a ***N by 1*** integer array that indicates the location of each diagonal term. If the stiffness matrix exceeds the high-speed memory capacity of the computer, a block storage form of the algorithm exists. Therefore, the capacity of the solution method is governed by the low speed disk capacity of the computer. This solution method is presented in detail in Appendix C of this book.

12.8 UNDAMPED HARMONIC RESPONSE

{ XE "Harmonic Loading" }The most common and very simple type of dynamic loading is the application of steady-state harmonic loads of the following form:

$$\mathbf{F}(t) = \mathbf{f} \sin(\overline{\omega} t) \tag{12.6}$$

The node point distribution of all static load patterns, $\mathbf{f}$, which are not a function of time, and the frequency of the applied loading, $\overline{\varpi}$, are user specified. Therefore, for the case of zero damping, the exact node point equilibrium equations for the structural system are:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}\sin(\overline{\varpi}t) \tag{12.7}$$

The exact steady-state solution of this equation requires that the node point displacements and accelerations are given by:

$$\mathbf{u}(\mathrm{t}) = \mathbf{v}\sin(\overline{\varpi}t), \qquad \ddot{\mathbf{u}}(\mathrm{t}) = -\,\mathbf{v}\,\overline{\varpi}^2\sin(\overline{\varpi}t) \tag{12.8}$$

Therefore, the harmonic node point response amplitude is given by the solution of the following set of linear equations:

$$[\mathbf{K} - \overline{\varpi}^2\mathbf{M}]\mathbf{v} = \mathbf{f} \qquad \text{or} \qquad \overline{\mathbf{K}}\mathbf{v} = \mathbf{f} \tag{12.9}$$

It is of interest to note that the normal solution for static loads is nothing more than a solution of this equation for zero frequency for all loads. It is apparent that the computational effort required for the calculation of undamped steady-state response is almost identical to that required by a static load analysis. Note that it is not necessary to evaluate mode shapes or frequencies to solve for this very common type of loading. The resulting node point displacements and member forces vary as $\sin(\overline{\varpi}t)$. However, other types of loads that do not vary with time, such as dead loads, must be evaluated in a separate computer run.

12.9 UNDAMPED FREE VIBRATIONS

{ XE "Undamped Free Vibration" }Most structures are in a continuous state of dynamic motion because of random loading such as wind, vibrating equipment, or human loads. These small ambient vibrations are normally near the natural frequencies of the structure and are terminated by energy dissipation in the real structure. However, special instruments attached to the structure can easily measure the motion. Ambient vibration field tests are often used to calibrate computer models of structures and their foundations.

After all external loads have been removed from the structure, the equilibrium equation, which governs the undamped free vibration of a typical displaced shape **v,** is:

$$\mathbf{M}\ddot{\mathbf{v}} + \mathbf{K}\mathbf{v} = \mathbf{0} \tag{12.10}$$

{ XE "Energy:Mechanical Energy" }{ XE "Mechanical Energy" }At any time, the displaced shape **v** may be a natural mode shape of the system, or any combination of the natural mode shapes. However, it is apparent the total energy within an undamped free vibrating system is a constant with respect to time. The sum of the kinetic energy and strain energy at all points in time is a constant that is defined as the ***mechanical energy*** of the dynamic system and calculated from:

$$E_M = \frac{1}{2}\dot{\mathbf{v}}^{\mathbf{T}}\mathbf{M}\dot{\mathbf{v}} + \frac{1}{2}\mathbf{v}^{\mathbf{T}}\mathbf{K}\mathbf{v} \tag{12.11}$$

12.10 SUMMARY

Dynamic analysis of three-dimensional structural systems is a direct extension of static analysis. The elastic stiffness matrices are the same for both dynamic and static analysis. It is only necessary to lump the mass of the structure at the joints. The addition of inertia forces and energy dissipation forces will satisfy dynamic equilibrium. The dynamic solution for steady state harmonic loading, without damping, involves the same numerical effort as a static solution. Classically, there are many different mathematical methods to solve the dynamic equilibrium equations. However, it will later be shown in this book that the majority of both linear and nonlinear systems can be solved with one numerical method.

Energy is fundamental in dynamic analysis. At any point in time, the external work supplied to the system must be equal to the sum of the kinetic and strain energy plus the energy dissipated in the system.

{ XE "Energy:Zero Strain Energy" }It is my opinion, with respect to earthquake resistant design, that we should try to minimize the mechanical energy in the structure. It is apparent that a rigid structure will have only kinetic energy and zero strain energy. On the other hand, a completely base isolated structure will have zero kinetic energy and zero strain energy. A structure cannot fail if it has zero strain energy.

12.11 REFERENCES

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