

10.

SHELL ELEMENTS

All Shell Elements Are Approximate and a Special Case of Three-Dimensional Elasticity

10.1 INTRODUCTION

{ XE "Shell Elements" }The use of classical thin shell theory for problems of arbitrary geometry leads to the development of higher order differential equations that, in general, can only be solved approximately using the numerical evaluation of infinite series. Therefore, a limited number of solutions exist only for shell structures with simple geometric shapes. Those solutions provide an important function in the evaluation of the numerical accuracy of modern finite element computer programs. However, for the static and dynamic analysis of shell structures of arbitrary geometry, which interact with edge beams and supports, the finite element method provides the only practical approach at this time.

Application of the finite element method for the analysis of shell structures requires that the user have an understanding of the approximations involved in the development of the elements. In the previous two chapters, the basic theory of plate and membrane elements has been presented. In this book, both the plate and membrane elements were derived as a special case of three-dimensional elasticity theory, in which the approximations are clearly stated. Therefore, using those elements for the analysis of shell structures involves the introduction of very few new approximations.

{ XE "Arch Dam" }Before analyzing a structure using a shell element, one should always consider the direct application of three-dimensional solids

to model the structure. For example, consider the case of a three-dimensional arch dam. The arch dam may be thin enough to use shell elements to model the arch section with six degrees-of-freedom per node; however, modeling the foundation requires the use of solid elements. One can introduce constraints to connect the two element types together. However, it is simpler and more accurate to use solid elements, with incompatible modes, for both the dam and foundation. For that case, only one element in the thickness direction is required, and the size of the element used should not be greater than two times the thickness. Because one can now solve systems of over one thousand elements within a few minutes on a personal computer, this is a practical approach for many problems.

10.2 A SIMPLE QUADRILATERAL SHELL ELEMENT

{ XE "Quadrilateral Element" } The two-dimensional plate bending and membrane elements presented in the previous two chapters can be combined to form a four-node shell element as shown in Figure 10.1.

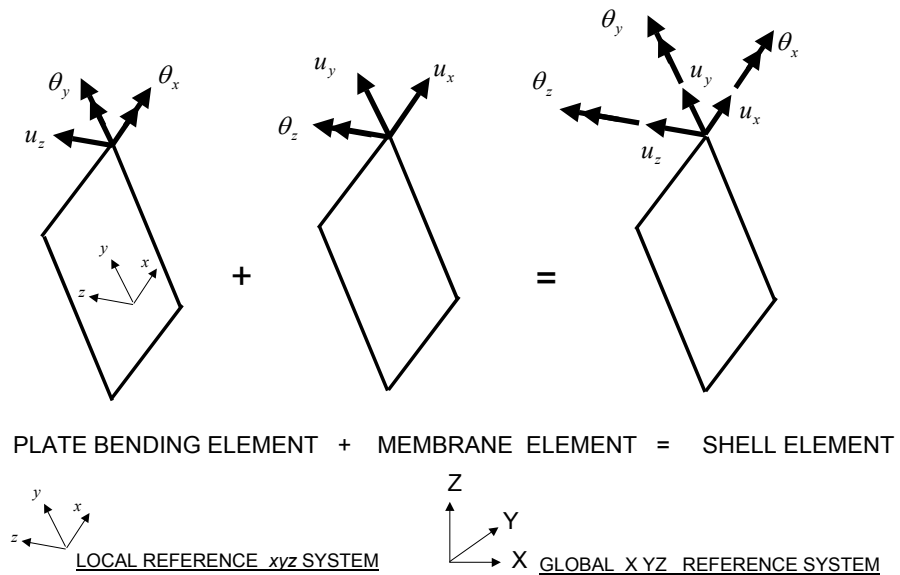


Figure 10.1 Formation of Flat Shell Element

It is only necessary to form the two element stiffness matrices in the local xyz system. The 24 by 24 local element stiffness matrix, Figure 10.1, is then transformed to the global XYZ reference system. The shell element stiffness and loads are then added using the direct stiffness method to form the global equilibrium equations.

Because plate bending (DSE) and membrane elements, in any plane, are special cases of the three-dimensional shell element, only the shell element needs to be programmed. This is the approach used in the SAP2000 program. As in the case of plate bending, the shell element has the option to include transverse shearing deformations.

10.3 MODELING CURVED SHELLS WITH FLAT ELEMENTS

{ XE "Arbitrary Shells" } Flat quadrilateral shell elements can be used to model most shell structures if all four nodes can be placed at the mid-thickness of the shell. However, for some shells with double curvature this may not be possible. Consider the shell structure shown in Figure 10.2.

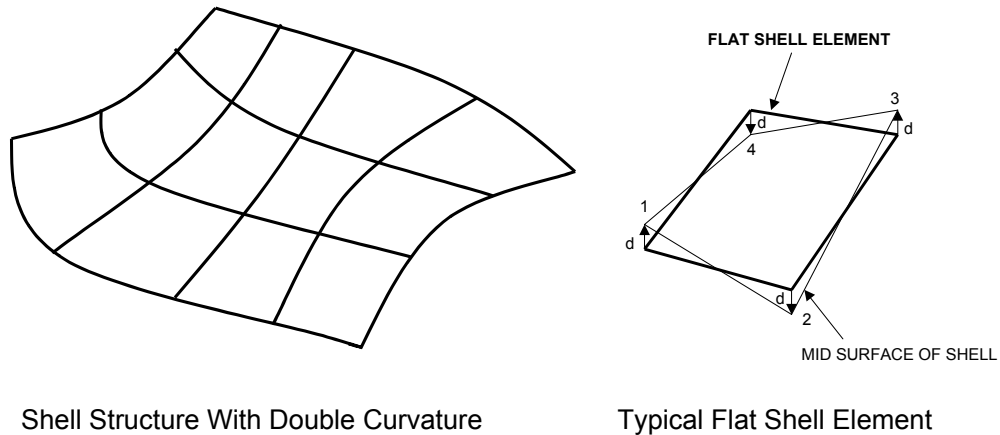


Figure 10.2 Use of Flat Elements to Model Arbitrary Shells

The four input points 1, 2 3 and 4 that define the element are located on the mid-surface of the shell, as shown in Figure 10.2. The local xyz coordinate system is defined by taking the cross product of the diagonal vectors. Or, $V_z = V_{1-3} V_{2-4}$. The distance vector d is normal to the flat element and is between the flat element node points and input node points at the mid-surface of the shell and is calculated from:

$$d = \pm \frac{z_1 + z_3 - z_2 - z_4}{2} \quad (10.1)$$

For most shells, this offset distance is zero and the finite element nodes are located at the mid-surface nodes. However, if the distance d is not zero, the flat element stiffness must be modified before transformation to the global XYZ reference system. It is very important to satisfy force equilibrium at the mid-surface of the shell structure. This can be accomplished by a transformation of the flat element stiffness matrix to the mid-surface locations by applying the following displacement transformation equation at each node:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}_n = \begin{bmatrix} 1 & 0 & 0 & 0 & -d & 0 \\ 0 & 1 & 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}_s \quad (10.2)$$

Physically, this is stating that the flat element nodes are rigidly attached to the mid-surface nodes. It is apparent that as the elements become smaller, the distance d approaches zero and the flat element results will converge to the shell solution.

10.4 TRIANGULAR SHELL ELEMENTS

{ XE "Triangular Elements" } It has been previously demonstrated that the triangular plate-bending element, with shearing deformations, produces excellent results. However, the triangular membrane element with drilling rotations tends to lock, and great care must be practiced in its application. Because any geometry can be modeled using quadrilateral elements, the use of the triangular element presented in this book can always be avoided.

10.5 USE OF SOLID ELEMENTS FOR SHELL ANALYSIS

The eight-node solid element with incompatible modes can be used for thick shell analysis. The cross-section of a shell structure modeled with eight-node solid elements is shown in Figure 10.3.

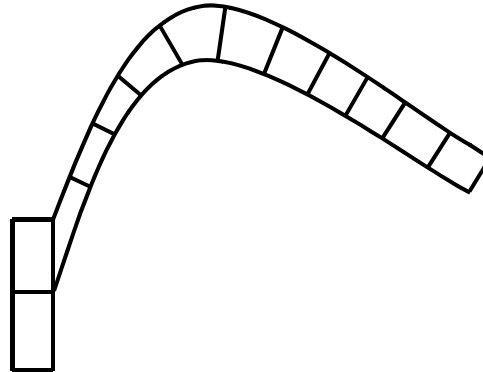


Figure 10.3 Cross-Section of Thick Shell Structure Modeled with Solid Elements

Note that there is no need to create a reference surface when solid elements are used. As in the case of any finite element analysis, more than one mesh must be used, and statics must be checked to verify the model, the theory and the computer program.

10.6 ANALYSIS OF THE SCORDELIS-LO BARREL VAULT

{ XE "Scordelis-Lo Barrel Vault" }The Scordelis-Lo barrel vault is a classical test problem for shell structures [1,2]. The structure is shown in Figure 10.4, with one quadrant modeled with a 4 by 4 shell element mesh. The structure is subjected to a factored gravity load in the negative z-direction. The maximum vertical displacement is 0.3086 ft. and mid-span moment is 2,090 lb. ft.

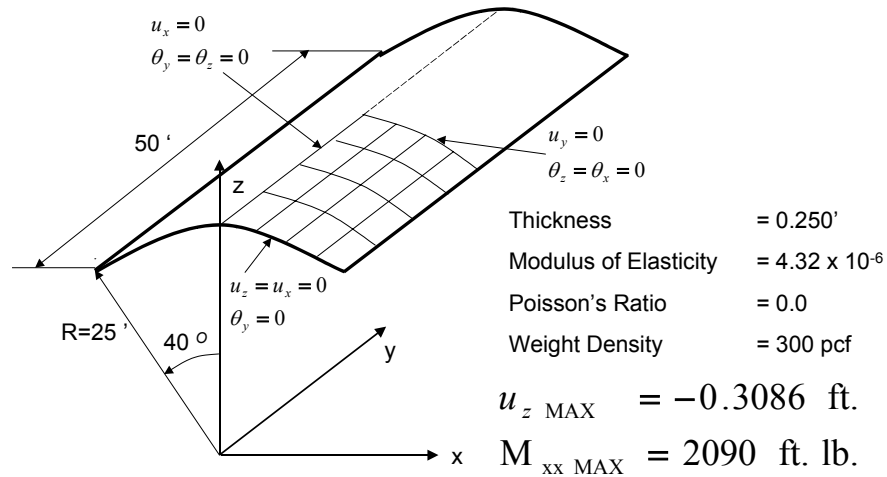


Figure 10.4 Scordelis-Lo Barrel Vault Example

To illustrate the convergence and accuracy of the shell element presented in this chapter, two meshes, with and without shearing deformations, will be presented. The results are summarized in Table 10.1.

Table 10.1 Result of Barrel Shell Analysis

	Theoretical	4 x4 DKE	4 x4 DSE	8 x 8 DKE	8 x 8 DSE
Displacement	0.3086	0.3173	0.3319	0.3044	0.3104
Moment	2090	2166	2252	2087	2113

{ XE "Plate Bending Elements:DSE" } One notes that the DSE tends to be more flexible than the DKE formulation. From a practical viewpoint, both elements yield excellent results. It appears that both will converge to almost the same result for a very fine mesh. Because of local shear deformation at the curved pinned edge, one would expect DSE displacement to converge to a slightly larger, and more correct, value.

10.7 HEMISPHERICAL SHELL EXAMPLE

{ XE "Kirchhoff Approximation" } The hemispherical shell shown in Figure 10.5 was proposed as a standard test problem for elements based on the Kirchhoff thin shell theory [1].

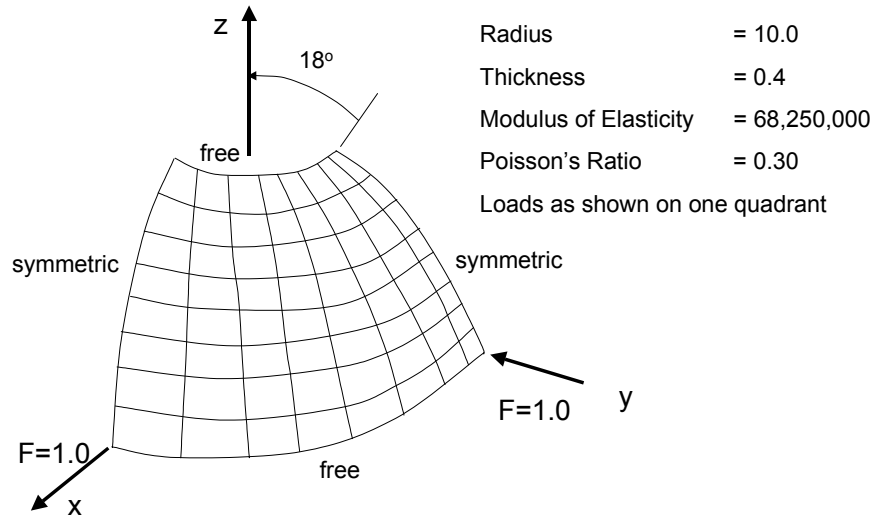


Figure 10.5 Hemispherical Shell Example

The results of the analyses using the DKE and DSE are summarized in Table 10.2. Because the theoretical results are based on the Kirchhoff approximation, the DKE element produces excellent agreement with the theoretical solution. The DSE results are different. Because the theoretical solution under a point load does not exist, the results using the DSE approximation are not necessarily incorrect.

Table 10.2 Result of Hemispherical Shell Analysis

	Theoretical	8 x 8 DKE	8 x 8 DSE
Displacement	0.094	0.0939	0.0978
Moment	-----	1.884	2.363

It should be emphasized that it is physically impossible to apply a point load to a real structure. All real loads act on a finite area and produce finite stresses. The point load, which produces infinite stress, is a mathematical definition only and cannot exist in a real structure.

10.8 SUMMARY

It has been demonstrated that the shell element presented in this book is accurate for both thin and thick shells. It appears that one can use the DSE approximation for all shell structures. The results for both displacements and moment appear to be conservative when compared to the DKE approximation.

10.9 REFERENCES

1. MacNeal, R. H. and R. C. Harder. 1985. "A Proposed Standard Set to Test Element Accuracy, Finite Elements in Analysis and Design." Vol. 1 (1985). pp. 3-20.
2. Scordelis, A. C. and K. S. Lo. 1964. "Computer Analysis of Cylinder Shells," *Journal of American Concrete Institute*. Vol. 61. May.